Anisotropy Corrections to the Valence Band in the Magnetic Field

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The energy levels for the holes in the valence semi-conductors in a constant magnetic field are considered on the basis of Luttinger's theory. The corrections due to the anisotropy of the energy bands and to the nonzero momentum component along the magnetic field are computed by the second-order perturbation calculation and are found to be generally small.

I. THE HAMILTONIAN MATRIX

THE general form of the Hamiltonian for holes in the valence semiconductors in the presence of a constant magnetic field **H** has been given by Luttinger.[1] It is impossible to solve the Schrödinger equation for an arbitrary direction of the magnetic field. Confining the calculations to the case when the vector **H** lies in the (110) plane, one can make a transformation of the components of the momentum operator:

$$k_x=(1/\sqrt{2})(k_1c-k_2+k_3s),$$
$$k_y=(1/\sqrt{2})(k_1c+k_2+k_3s),$$
$$k_z=-k_1s+k_3c,$$

where $s=\sin\theta$, $c=\cos\theta$, and θ is the angle between the vector **H** and the z axis. Making a corresponding transformation of the angular momentum matrices, the J's, we get the following form of the Hamiltonian:

$$D=D_0+D_1+D_3+D_4. \tag{1}$$

The Hamiltonian D, measured in units of $\hbar\omega=\hbar(eH/mc)$, is determined by the cyclotron resonance parameters γ_1, γ_2, and γ_3. Writing D in the form (1), we neglect the spin-orbit coupling, i.e., we take $q=0$. D_0 and D_1 are given by formula (81) in the Luttinger paper.[1] D_3 and D_4 contain terms dependent on the component k_3 and vanish when $k_3=0$.

The explicit matrix representations of D_3 and D_4 are

$$D_3=\begin{Vmatrix} w_1 & 0 & -w_2 & 0 \\ 0 & w_3 & 0 & w_2 \\ -w_2^\dagger & 0 & w_3 & 0 \\ 0 & w_2^\dagger & 0 & w_1 \end{Vmatrix}, \tag{2}$$

where

$$w_1=\tfrac{1}{2}k^2(\gamma_1-2\gamma'),$$
$$w_2=\sqrt{6}\gamma'''ka,$$
$$w_3=\tfrac{1}{2}k^2(\gamma_1+2\gamma'),$$
$$\gamma'=\tfrac{1}{4}[(3c^2-1)^2\gamma_2+3s^2(3c^2+1)\gamma_3],$$
$$\gamma'''=\tfrac{1}{2}[(1+2c^2-3c^4)\gamma_2+(1-2c^2+3c^4)\gamma_3],$$

and

$$k\equiv(eH/\hbar c)^{-1/2}k_3,$$

$$D_4=\sqrt{3}\mu\begin{Vmatrix} t_1 & t_2 & t_3 & 0 \\ t_2^\dagger & -t_1 & 0 & -t_3 \\ t_3^\dagger & 0 & -t_1 & t_2 \\ 0 & -t_3^\dagger & t_2^\dagger & t_1 \end{Vmatrix}, \tag{3}$$

where

$$t_1=-(3/2)^{1/2}ksc(3c^2-1)(a^\dagger+a),$$
$$t_2=\tfrac{1}{2}k^2s^2(3c^2-1)+(1/\sqrt{2})ksc[(3c^2-5)a^\dagger+(3c^2-1)a],$$
$$t_3=k^2sc(3c^2-1)-\sqrt{2}k(1-4c^2+3c^4)a^\dagger,$$

and

$$\mu=(\gamma_3-\gamma_2)/2.$$

II. PERTURBATION CALCULATION

The general solution of the equation

$$D\psi=\epsilon\psi,$$

has been given by Evtuhov.[2] In his method, the wave function is developed into an infinite series of harmonic oscillator functions. Here, we present a simpler but less general method of solution of this problem by the second-order perturbation theory.

Treating the anisotropy, proportional to μ, as a perturbation, we now compute the corrections to the energy levels (Roth, Lax, and Zwerdling[3] and Goodman[4]). The unperturbed state of the system is described by the function $\psi_{qn}(k)$ given by the solution of the equation

$$(D_0+D_3)\psi_{qn}(k)=\epsilon_{qn}(k)\psi_{qn}(k). \tag{4}$$

The function $\psi_{qn}(k)$ has the form

$$\psi_{qn}(k)=\begin{Vmatrix} b_{qn}{}^1(k)u_{n-2} \\ b_{qn}{}^2(k)u_n \\ b_{qn}{}^3(k)u_{n-1} \\ b_{qn}{}^4(k)u_{n+1} \end{Vmatrix}, \tag{5}$$

for every value of n, if we take

for $n\leqslant -2$, all $b_{qn}(k)=0$;
for $n=-1$, $b_{q,-1}{}^1=b_{q,-1}{}^2=b_{q,-1}{}^3=0$;
for $n=0$, $b_{q0}{}^1=b_{q0}{}^3=0$;
for $n=1$, $b_{q1}{}^1=0$;
for $n\geqslant 2$, all $b_{qn}(k)\neq 0$.

After substitution (5) into (4), we get the following system of the homogeneous equations for the coeffi-

[1] J. M. Luttinger, Phys. Rev. **102**, 1030 (1956).

[2] V. Evtuhov, Phys. Rev. **125**, 1869 (1962).

[3] L. M. Roth, B. Lax, and S. Zwerdling, Phys. Rev. **114**, 90 (1959).

[4] R. R. Goodman, Phys. Rev. **122**, 397 (1961).

cients $b_{qn}(k)$:

$$
\begin{aligned}
&[(\gamma_1+\gamma')(n-\tfrac{3}{2})+\tfrac{3}{2}\kappa+\tfrac{1}{2}k^2(\gamma_1-2\gamma')-\epsilon_{qn}]b_{qn}{}^1\\
&\quad-[3n(n-1)]^{1/2}\gamma''b_{qn}{}^2-[6(n-1)]^{1/2}\gamma'''kb_{qn}{}^3=0,\\
&-[3n(n-1)]^{1/2}\gamma''b_{qn}{}^1+[(\gamma_1-\gamma')(n+\tfrac{1}{2})-\tfrac{1}{2}\kappa\\
&\quad+\tfrac{1}{2}k^2(\gamma_1+2\gamma')-\epsilon_{qn}]b_{qn}{}^2+[6(n+1)]^{1/2}\gamma'''kb_{qn}{}^4=0,\\
&-[6(n-1)]^{1/2}\gamma'''kb_{qn}{}^1+[(\gamma_1-\gamma')(n-\tfrac{1}{2})+\tfrac{1}{2}\kappa \qquad (6)\\
&\quad+\tfrac{1}{2}k^2(\gamma_1+2\gamma')-\epsilon_{qn}]b_{qn}{}^3-[3n(n+1)]^{1/2}\gamma''b_{qn}{}^4=0,\\
&[6(n+1)]^{1/2}\gamma'''kb_{qn}{}^2-[3n(n+1)]^{1/2}\gamma''b_{qn}{}^3\\
&\quad+[(\gamma_1+\gamma')(n+\tfrac{3}{2})-\tfrac{3}{2}\kappa+\tfrac{1}{2}k^2(\gamma_1-2\gamma')-\epsilon_{qn}]b_{qn}{}^4=0.
\end{aligned}
$$

Here

$$\gamma''=\tfrac{1}{8}[(3-2c^2+3c^4)\gamma_2+(5+2c^2-3c^4)\gamma_3].$$

The quantity n corresponds to the Landau magnetic quantum number; the quantity q numbers the solutions of the secular Eq. (7).

From the normalization of the function ψ_{qn}, we get the following condition for the coefficients $b_{qn}(k)$:

$$(b_{qn}{}^1)^2+(b_{qn}{}^2)^2+(b_{qn}{}^3)^2+(b_{qn}{}^4)^2=1.$$

The eigenvalues ϵ_{qn} are given by the solution of the secular equation (compare Wallis and Bowlden,[5] Suffczyński,[6] Elliott, McLean, and Macfarlane[7]):

$$
\begin{vmatrix}
(\gamma_1+\gamma')(n-\frac{3}{2})+\frac{3}{2}\kappa+\frac{1}{2}k^2(\gamma_1-2\gamma')-\epsilon_{qn} & -[3n(n-1)]^{1/2}\gamma'' & -[6(n-1)]^{1/2}k\gamma''' & 0\\
-[3n(n-1)]^{1/2}\gamma'' & (\gamma_1-\gamma')(n+\frac{1}{2})-\frac{1}{2}\kappa+\frac{1}{2}k^2(\gamma_1+2\gamma')-\epsilon_{qn} & 0 & [6(n+1)]^{1/2}k\gamma'''\\
-[6(n-1)]^{1/2}k\gamma''' & 0 & (\gamma_1-\gamma')(n-\frac{1}{2})+\frac{1}{2}\kappa+\frac{1}{2}k^2(\gamma_1+2\gamma')-\epsilon_{qn} & -[3n(n+1)]^{1/2}\gamma''\\
0 & [6(n+1)]^{1/2}k\gamma''' & -[3n(n+1)]^{1/2}\gamma'' & (\gamma_1+\gamma')(n+\frac{3}{2})-\frac{3}{2}\kappa+\frac{1}{2}k^2(\gamma_1-2\gamma')-\epsilon_{qn}
\end{vmatrix}=0. \qquad (7)
$$

The perturbation is described by the Hamiltonian

$$D'=D_1+D_4,$$

which is proportional to μ. Corrections to the energy $\epsilon_{qn}(k)$ can be obtained from the standard formulas of the perturbation theory. The first order correction vanishes identically because D_1 and D_4 have no diagonal matrix elements in the D_0+D_3 representation.

The second-order correction is

$$\epsilon_{qn}{}^{(2)}(k)=\sum_{m\neq n}\sum_{p}\frac{|\langle n,q,k|D_1+D_4|m,p,k\rangle|^2}{\epsilon_{qn}(k)-\epsilon_{pm}(k)}. \qquad (8)$$

The summation over the intermediate states goes over all ladders and all values of n which combine with the level under consideration.

The only nonvanishing matrix elements of the Hamiltonian parts D_1 and D_4 are:

$$
\begin{aligned}
&\langle n,q,k|D_1|n+4,p,k\rangle\\
&\quad=\tfrac{1}{4}\sqrt{3}\mu(3c^2-1)(c^2-3)\{[(n+1)(n+2)]^{1/2}\\
&\qquad\times b_{p,n+4}{}^1b_{qn}{}^2+[(n+2)(n+3)]^{1/2}b_{p,n+4}{}^3b_{qn}{}^4\},\\
&\langle n,q,k|D_1|n+3,p,k\rangle\\
&\quad=\tfrac{1}{2}\sqrt{3}\mu sc(3c^2-5)\{[n(n+1)]^{1/2}b_{p,n+3}{}^1b_{qn}{}^3\\
&\qquad-[(n+2)(n+3)]^{1/2}b_{p,n+3}{}^2b_{qn}{}^4\},\\
&\langle n,q,k|D_1|n+2,p,k\rangle\\
&\quad=\tfrac{1}{2}\sqrt{3}\mu s^2(3c^2-1)\{\tfrac{1}{2}[3n(n-1)]^{1/2}b_{p,n+2}{}^1b_{qn}{}^1\\
&\qquad-\tfrac{1}{2}[3(n+1)(n+2)]^{1/2}b_{p,n+2}{}^2b_{qn}{}^2\\
&\qquad-\tfrac{1}{2}[3n(n+1)]^{1/2}b_{p,n+2}{}^3b_{qn}{}^3\\
&\qquad+\tfrac{1}{2}[3(n+2)(n+3)]^{1/2}b_{p,n+2}{}^4b_{qn}{}^4\\
&\qquad-(n+\tfrac{1}{2})b_{p,n+2}{}^1b_{qn}{}^2-(n+\tfrac{3}{2})b_{p,n+2}{}^3b_{qn}{}^4\},
\end{aligned}
$$

$$
\begin{aligned}
&\langle n,q,k|D_1|n+1,p,k\rangle\\
&\quad=\tfrac{1}{2}\sqrt{3}\mu sc(3c^2-1)\{(2n-1)b_{p,n+1}{}^1b_{qn}{}^3\\
&\qquad-(2n+3)b_{p,n+1}{}^2b_{qn}{}^4+[n(n-1)]^{1/2}b_{p,n+1}{}^3b_{qn}{}^1\\
&\qquad-[(n+1)(n+2)]^{1/2}b_{p,n+1}{}^4b_{qn}{}^2\},\\
&\langle n,q,k|D_1|n-1,p,k\rangle\\
&\quad=\tfrac{1}{2}\sqrt{3}\mu sc(3c^2-1)\{[(n-1)(n-2)]^{1/2}b_{p,n-1}{}^1b_{qn}{}^3\\
&\qquad-[n(n-1)]^{1/2}b_{p,n-1}{}^2b_{qn}{}^4+(2n-3)b_{p,n-1}{}^3b_{qn}{}^1\\
&\qquad-(2n+1)b_{p,n-1}{}^4b_{qn}{}^2\},\\
&\langle n,q,k|D_1|n-2,p,k\rangle\\
&\quad=\tfrac{1}{2}\sqrt{3}\mu s^2(3c^2-1)\{\tfrac{1}{2}[3(n-2)(n-3)]^{1/2}b_{p,n-2}{}^1b_{qn}{}^1\\
&\qquad-\tfrac{1}{2}[3n(n-1)]^{1/2}b_{p,n-2}{}^2b_{qn}{}^2\\
&\qquad-\tfrac{1}{2}[3(n-1)(n-2)]^{1/2}b_{p,n-2}{}^3b_{qn}{}^3\\
&\qquad+\tfrac{1}{2}[3n(n+1)]^{1/2}b_{p,n-2}{}^4b_{qn}{}^4\\
&\qquad-(n-\tfrac{3}{2})b_{p,n-2}{}^2b_{qn}{}^1-(n-\tfrac{1}{2})b_{p,n-2}{}^4b_{qn}{}^2\},\\
&\langle n,q,k|D_1|n-3,p,k\rangle\\
&\quad=\tfrac{1}{2}\sqrt{3}\mu sc(3c^2-5)\{[(n-2)(n-3)]^{1/2}b_{p,n-3}{}^3b_{qn}{}^1\\
&\qquad-[n(n-1)]^{1/2}b_{p,n-3}{}^4b_{qn}{}^2\},\\
&\langle n,q,k|D_1|n-4,p,k\rangle\\
&\quad=\tfrac{1}{4}\sqrt{3}\mu(3c^2-1)(c^2-3)\{[(n-2)(n-3)]^{1/2}\\
&\qquad\times b_{p,n-4}{}^2b_{qn}{}^1+[(n-1)(n-2)]^{1/2}b_{p,n-4}{}^4b_{qn}{}^3\},\\
&\langle n,q,k|D_4|n+3,p,k\rangle\\
&\quad=(\tfrac{3}{2})^{1/2}\mu ksc(3c^2-5)[(n+1)^{1/2}b_{p,n+3}{}^1b_{qn}{}^2\\
&\qquad+(n+2)^{1/2}b_{p,n+3}{}^3b_{qn}{}^4],
\end{aligned}
$$

[5] R. F. Wallis and H. J. Bowlden, Phys. Rev. **118**, 456 (1960).
[6] M. Suffczyński, Proc. Phys. Soc. (London) **77**, 1042 (1961).
[7] R. J. Elliott, T. P. McLean, and G. G. Macfarlane, Proc. Phys. Soc. (London) **72**, 553 (1958).

$$\langle n,q,k|D_4|n+2,p,k\rangle$$
$$=\sqrt{3}\mu\{\tfrac{1}{2}k^2s^2(3c^2-1)(b_{p,n+2}{}^1b_{qn}{}^2+b_{p,n+2}{}^3b_{qn}{}^4)$$
$$-\sqrt{2}k(1-4c^2+3c^4)[n^{1/2}b_{p,n+2}{}^1b_{qn}{}^3$$
$$-(n+2)^{1/2}b_{p,n+2}{}^2b_{qn}{}^4]\},$$
$$\langle n,q,k|D_4|n+1,p,k\rangle$$
$$=-\sqrt{3}\mu sc(3c^2-1)\{k^2(b_{p,n+1}{}^1b_{qn}{}^3-b_{p,n+1}{}^2b_{qn}{}^4)$$
$$+(1/\sqrt{2})k[(3(n-1))^{1/2}b_{p,n+1}{}^1b_{qn}{}^1$$
$$-(3(n+1))^{1/2}b_{p,n+1}{}^2b_{qn}{}^2-(3n)^{1/2}b_{p,n+1}{}^3b_{qn}{}^3$$
$$+(3(n+2))^{1/2}b_{p,n+1}{}^4b_{qn}{}^4-n^{1/2}b_{p,n+1}{}^1b_{qn}{}^2$$
$$-(n+1)^{1/2}b_{p,n+1}{}^3b_{qn}{}^4\},$$
$$\langle n,q,k|D_4|n-1,p,k\rangle$$
$$=-\sqrt{3}\mu sc(3c^2-1)\{k^2[b_{p,n-1}{}^3b_{qn}{}^1-b_{p,n-1}{}^4b_{qn}{}^2]$$
$$+(1/\sqrt{2})k[(3(n-2))^{1/2}b_{p,n-1}{}^1b_{qn}{}^1$$
$$-(3n)^{1/2}b_{p,n-1}{}^2b_{qn}{}^2-(3(n-1))^{1/2}b_{p,n-1}{}^3b_{qn}{}^3$$
$$+(3(n+1))^{1/2}b_{p,n-1}{}^4b_{qn}{}^4$$
$$-(n-1)^{1/2}b_{p,n-1}{}^2b_{qn}{}^1-n^{1/2}b_{p,n-1}{}^4b_{qn}{}^3\},$$
$$\langle n,q,k|D_4|n-2,p,k\rangle$$
$$=\sqrt{3}\mu\{\tfrac{1}{2}k^2s^2(3c^2-1)(b_{p,n-2}{}^2b_{qn}{}^1+b_{p,n-2}{}^4b_{qn}{}^3)$$
$$-\sqrt{2}k(1-4c^2+3c^4)[(n-2)^{1/2}b_{p,n-2}{}^3b_{qn}{}^1$$
$$-n^{1/2}b_{p,n-2}{}^4b_{qn}{}^2]\},$$
$$\langle n,q,k|D_4|n-3,p,k\rangle$$
$$=(\tfrac{3}{2})^{1/2}\mu ksc(3c^2-5)\{(n-2)^{1/2}b_{p,n-3}{}^2b_{qn}{}^1$$
$$+(n-1)^{1/2}b_{p,n-3}{}^4b_{qn}{}^3\}. \quad (9)$$

The other matrix elements of D_1 and D_4 vanish because the eigenfunctions u_n of a harmonic oscillator are orthogonal for different n's.

III. NUMERICAL RESULTS

In order to estimate the corrections due to anisotropy in valence semiconductors, a numerical calculations has been made for germanium and silicon for the angle θ equal to 0°, 45°, 90°. In these calculations, the following values of constants γ_1, γ_2, γ_3, and κ which appear in the above formulas are used: for germanium (Roth, Lax, and Zwerdling)[3]

$$\gamma_1=13.1,\quad \gamma_2=4.15,\quad \gamma_3=5.5,\quad \kappa=3.23,$$

for silicon (Stickler, Zeiger, and Heller)[8]

$$\gamma_1=4.22,\quad \gamma_2=0.50,\quad \gamma_3=1.35,\quad \kappa=-0.39.$$

The k dependence of the unperturbed energy levels is obtained from the solution of the Eq. (7). This dependence is nearly parabolic for the two light-hole ladders except of the lowest Landau numbers n for all considered directions of the magnetic field. More interesting is the k dependence of the heavy-hole levels. In one of the ladders, namely in the ϵ_1^- ladder, there appear minima for values of k different from zero. For $\theta=0°$, the distance of these minima from the center of the Brillouin zone increases as the Landau number n increases, while for $\theta=45°$ and $\theta=90°$, this distance decreases with increasing n. Ths shape of these levels is similar to those obtained by Wallis and Bowlden[5] and Evtuhov.[2]

[8] J. J. Stickler, H. J. Zeiger, and G. S. Heller, Phys. Rev. **129**, 1077 (1962).

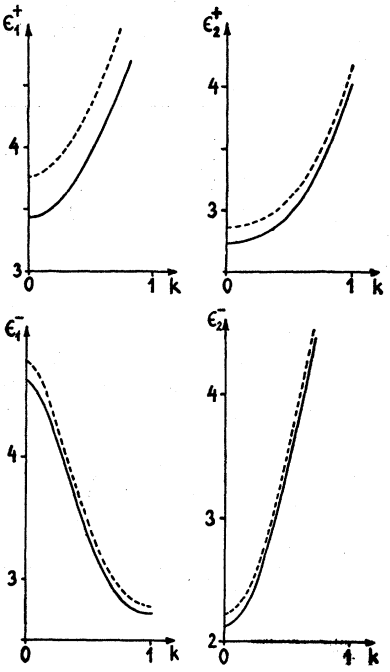

FIG. 1. The corrected lowest levels in four ladders in Ge for the angle $\theta=0°$. The full lines show corrected levels; the dashed lines show uncorrected levels.

TABLE I. Cyclotron resonance mass ratios (m_c/m) in Ge for the angle $\theta=0°$.

n	ϵ_1^+	ϵ_2^+	ϵ_1^-	ϵ_2^-
−1	0.058			
0	0.047	0.112		
1	0.046	0.057		0.234
2	0.045	0.047	0.200	0.248
3	0.045	0.046	0.235	0.254
4	0.045	0.045	0.248	0.258

Both the curvature and the cyclotron resonance masses depend on the direction of the magnetic field even for unperturbed levels if we take different values for the parameters γ_2 and γ_3 in the Hamiltonian D_0+D_3. The cyclotron resonance masses of uncorrected levels, i.e., the reciprocals of the separation between n and $n+1$ levels, at $k=0$, are given in Tables I–VI for different values of the angle θ. In both considered

TABLE II. Cyclotron resonance mass ratios (m_c/m) in Ge for the angle $\theta=45°$.

n	ϵ_1^+	ϵ_2^+	ϵ_1^-	ϵ_2^-
−1	0.054			
0	0.045	0.130		
1	0.044	0.054		0.310
2	0.043	0.045	0.256	0.330
3	0.043	0.044	0.310	0.338
4	0.043	0.043	0.329	0.343

TABLE III. Cyclotron resonance mass ratios (m_c/m) in Ge for the angle $\theta=90°$.

n	ϵ_1^+	ϵ_2^+	ϵ_1^-	ϵ_2^-
−1	0.055			
0	0.045	0.126		
1	0.044	0.055		0.291
2	0.044	0.045	0.243	0.309
3	0.043	0.044	0.291	0.317
4	0.043	0.044	0.309	0.321

semiconductors, the cyclotron resonance masses of the light holes change slightly for different directions of the magnetic field. The departure from the uniform spacing for low n's occurs in both Ge and Si but is smaller and decreases more rapidly as n increases in Ge

TABLE IV. Cyclotron resonance mass ratios (m_c/m) in Si for the angle $\theta=0°$.

n	ϵ_1^+	ϵ_2^+	ϵ_1^-	ϵ_2^-
−1	0.212			
0	0.193	0.269		
1	0.183	0.231		0.335
2	0.178	0.204	0.283	0.353
3	0.175	0.189	0.318	0.365
4	0.174	0.181	0.342	0.372

than in Si. This is conceivable because the relative anisotropy $(\gamma_3-\gamma_2)/(\gamma_3+\gamma_2)$ is larger in Si than in Ge. The values of m_c/m are in accord with the experimental data quoted by Dresselhaus, Kip, and Kittel.[9] The

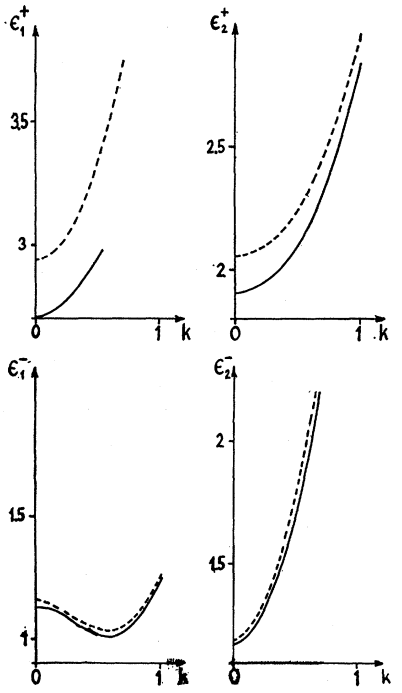

FIG. 2. The corrected lowest levels in four ladders in Si for the angle $\theta=0°$. The full lines show corrected levels; the dashed lines show uncorrected levels.

[9] G. Dresselhaus, A. F. Kip, and C. Kittel, Phys. Rev. **98**, 368 (1955).

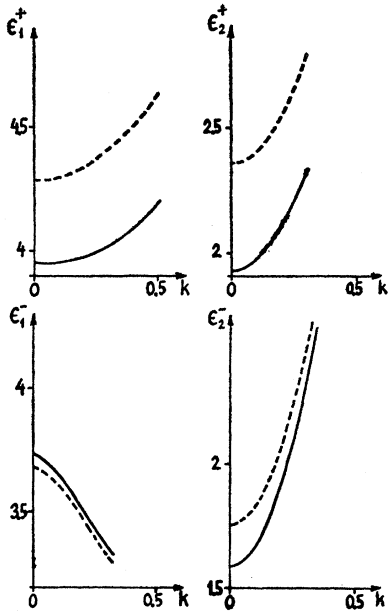

FIG. 3. The corrected lowest levels in four ladders in Ge for the angle $\theta=90°$. The full lines show corrected levels; the dashed lines show uncorrected levels.

cyclotron resonance masses of the heavy holes exhibit pronounced dependence on the direction of the magnetic field. The character of this dependence agrees with the experimental data but the magnitude of (m_c/m) is smaller by a few percent than the values given in cyclotron resonance measurements.[9]

TABLE V. Cyclotron resonance mass ratios (m_c/m) in Si for the angle $\theta=45°$.

n	ϵ_1^+	ϵ_2^+	ϵ_1^-	ϵ_2^-
−1	0.181			
0	0.166	0.342		
1	0.161	0.252		0.450
2	0.158	0.193	0.308	0.470
3	0.157	0.170	0.390	0.481
4	0.156	0.162	0.438	0.488

The knowledge of the dependence ϵ_{qn} on k permits us to estimate the extent of validity of the perturbation calculation. The second-order perturbation calculation is limited by the condition that in the perturbation formula the numerators must be smaller than the energy denominators. An analysis of the relative position of the energy levels reveals that this condition

TABLE VI. Cyclotron resonance mass ratios (m_c/m) in Si for the angle $\theta=90°$.

n	ϵ_1^+	ϵ_2^+	ϵ_1^-	ϵ_2^-
−1	0.187			
0	0.171	0.324		
1	0.165	0.248		0.420
2	0.162	0.196	0.298	0.439
3	0.161	0.175	0.368	0.451
4	0.160	0.167	0.411	0.457

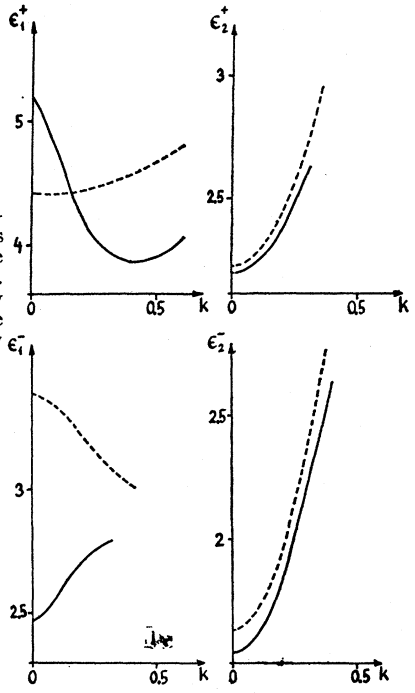

FIG. 4. The corrected lowest levels in four ladders in Ge for the angle $\theta=45°$. The full lines show corrected levels; the dashed lines show uncorrected levels.

is valid for small values of k, so small that they constitute a fraction (0.3–0.5) of a percent of the dimension of the Brillouin zone for magnetic fields as large as $H=100$ kG. The reason for such a limitation is a strong mixing of the levels of different ladders, especially in Si. The second-order corrections were calculated for the lowest energy levels. From Eqs. (9) it can easily be seen that the influence of the anisotropy upon the shape of the energy levels will be stronger for intermediate directions between the [001] and [$\bar{1}$10] directions because then all the matrix elements of the perturbation are different from zero. In the case when the vector **H** is in the [001] or [$\bar{1}$10] direction, taking account of the anisotropy does not change the shape of the energy levels. In both Ge and Si, the anisotropy corrections due to D_1+D_4 lower only the energy levels and slightly change their curvature. The corrected levels for these directions are plotted in Figs. 1–3. For the intermediate angle $\theta=45°$, the corrections were appreciable for the lowest levels in ϵ_1^+ and ϵ_1^- ladders (see Fig. 4). The large value of the matrix element

TABLE VII. Corrected cyclotron resonance mass ratios (m_c/m) for the lowest energy levels in Ge and Si for the angle $\theta=0°$.

	Ge	Si
$\epsilon_1^+(-1)$	0.054	0.276
$\epsilon_2^+(0)$	0.119	0.295
$\epsilon_1^-(2)$	0.210	0.291
$\epsilon_2^-(1)$	0.320	0.341

$\langle -1, 1, k|D_1+D_4|2, 3, k\rangle$ of the perturbation and the small separation between these levels are the cause for this modification. In Ge, the curvature of these levels varies not only in magnitude but also in sign, instead the spacing varies slightly. In Si, where the constant μ is smaller than in Ge, an analogous calculation gives a distinct change of magnitude of the curvature for these levels.

The corrected cyclotron resonance masses are given in Table VII. They differ from the uncorrected by a few percent. The only pronounced change in the ϵ_1^+ ladder in Si follows from the small separation between the $\epsilon_{1,-1}^+$ and $\epsilon_{2,4}^-$ levels even for $k=0$, so small that in the perturbation formula the numerator is larger than the energy denominator, i.e., the above mentioned condition of validity of the perturbation theory is not valid.

The latest measurements of the cyclotron resonance parameters γ_1, γ_2, γ_3 in Si (Hensel and Feher)[10] give for μ a value by 40% larger than that used in the present calculations. Because the anisotropy corrections are proportional to μ^2, it can be expected that the corrections for Si will be greater than those calculated here.

Practical implication which results from the present calculations is that the form of the ladders resulting from the matrix D_0+D_3 is entirely satisfactory for most applications.

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[10] J. C. Hensel and G. Feher, Phys. Rev. **129**, 1041 (1963).